

FACULTY OF ENGINEERING
B.E. I - Semester (AICTE) (Main) Examination, July 2021

Subject: Mathematica - I

Time: 2 Hours

Max. Marks: 70

- Note:** (i) First question is compulsory and answer any three questions from the remaining six questions.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1 Answer any four questions.

(4 x 4 = 16 Marks)

(a) Examine the convergence of the sequence $\{a_n\}$ where $a_n = 1 + (-1)^n$.

(b) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

(c) Using Lagrange mean value theorem, show that $|\cos b - \cos a| < |b - a|$.

(d) If $r(x, y) = x \cos y + e^x \sin y$, $x = t^2 + 1$, $y = t^2 + 1$, then find $\frac{dr}{dt}$ at $t = 0$.

(e) If $z = x^2 + y^2 + 3xyz$, then find $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$, $\frac{\partial^2 z}{\partial y^2}$.

(f) Find the area of the region bounded by the curves $x = y^2$, $x + y - 2 = 0$.

(g) If $z = u + y + zk$ then show that (i) $\nabla \cdot \vec{r} = 3$ (ii) $\nabla \times \vec{r} = 0$.

(3x18 = 54 Marks)

2 (a) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)}$.

(b) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(n!)^2}$.

3 (a) Find 'c' of Cauchy's mean value theorem for $f(x) = \log_e x$ and $g(x) = 1/x$ in the interval $[1, e]$.

(b) Find the radius of curvature at any point on the cardioid $r = a(1 - \cos \theta)$.

4 (a) If $z = f(x + ay) + \phi(x - ay)$, show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

(b) Expand $e^x \log(1+y)$ in powers of x and y up to the terms of third degree using Taylor's theorem.

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